Asymptotic Accuracy of the Equilibrium-Diffusion Approximation

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Abstract

The equilibrium-diffusion approximation takes two forms: one that relates to a static medium and one that relates to the non-relativistic radiation-hydrodynamics equations. Both of these forms have been previously derived via a formal asymptotic expansion procedure. For the case of a static medium, Morel has shown that the approximation is correct to first order rather than simply leading (zeroth) order, and used this result to investigate the asymptotic accuracy of several flux-limited diffusion theories. For the case of the radiation-hydrodynamics equations, Lowrie, Morel, and Hittinger demonstrated accuracy only to leading order. We extend their radiation-hydrodynamic analysis to show that the equilibrium-diffusion approximation is correct to first order. In addition, we use our results to investigate the accuracy of a grey P-1 approximation made in the comoving frame versus the accuracy of a grey P-1 approximation made in the lab frame. We stress that these are two fundamentally different approximations, i.e., one cannot be obtained from the other via a Lorentz transformation. Since in the equilibrium-diffusion limit the radiation intensity only becomes isotropic in the comoving frame, but not the lab frame, one might expect the lab-frame P-1 approximation to be flawed in this limit. However, our analysis shows that both approximations are correct to first order in this limit, which represents the highest order of accuracy that can be expected.